Abstract: In this paper, flexure hinges as the main part of the actuators frame are studied considering different profiles of flexure hinges and effective design parameters to introduce the best profile of the hinge based on the application that is needed.

For this purpose five different profiles (right circular, elliptical, corner-filleted, hyperbolic and parabolic) of flexure hinges have been considered and further studies have been carried out on flexure hinges based on some parameters such as accuracy, flexibility and the maximum stress in the hinge.

The amount of flexibility of the hinge determined based on strain energy, accuracy of hinge (amount of deviation from actual hinge) and stresses in hinge is achieved by invoking FEM analysis, we compare different profiles used in flexure hinges and the behavior of accuracy is studied with deflection variation.

Different flexure hinge profiles are used in a sample mechanism and amplification factor, natural frequency and accuracy of results are compared with FEM results. In order to verify the obtained results, the sample mechanism with corner-filleted flexure hinge with the fillet radius of 0.1 mm is manufactured and the FEM results are verified with measured displacement of this sample frame.

Keywords: flexure hinges, flexure profile, accuracy, flexibility, amplification factor, FEM

1. INTRODUCTION

Nowadays, usage of piezoelectric actuators has been increased significantly in precision positioning industries such as precision X-Y stages (Moriyama et al., 1988), piezoelectric motors (Ishida, 1992), active dampers, high-accuracy alignment devices for optical fibers (Galvagni, 1990), scanning tunneling microscopes, high-precision cameras, robotic micro-displacement mechanisms, antennas and valves. Because of the low displacement of piezoelectric actuators (only about 0.1% of its length), flexure hinges are used to magnify the piezostacks displacement amplitude considering their advantages such as designs requiring one-piece(monolithic) manufacturing, reduced weight, smoothness of movement, zero backlashes, no need of lubrication, and virtually infinite resolution.

Paros and Weisbord (1965), in their fundamental work, presented the design equations, both exact and simplified, for calculating the compliances (spring rates) of single-axis and two-axis circular cutout constant cross-section flexure hinges. Ragulskis et al. (1989) applied the static finite element analysis to one-quarter of circular flexure hinges in order to calculate their compliances. The analysis results were further used to formulate an existence criterion based on the deformation of an initially straight cross-section; this allowed specifying optimal flexure geometry for minimum bending stiffness. Lobontiu et al. (2001) developed an analytical model of corner-filleted flexure hinges that are incorporated into planar amplification mechanisms. Compliance factors were formulated that allow evaluating the rotation efficiency, precision of motion and stresses. A corner-filleted flexure hinge spans a design space that is limited by the right circular flexure and the simple beam, in terms of its compliance. Xu and King (1996) performed static finite element analysis of circular, corner-filleted and elliptic flexure hinges. The results revealed that the corner-filleted flexure is the most accurate in terms of motion, the elliptic flexure has less stress for the same displacement, while the right circular flexure is the stiffest. Ryu and Gweon (1997) modeled the motion errors that are induced by machining imperfections into a flexure hinge mechanism. Lobontiu et al. (2002) introduced the parabolic and hyperbolic flexure hinges for planar mechanisms, by using compliance closed-form solutions to characterize their performance in terms of flexibility, precision of motion and stresses.

At this work, to achieve the optimum amount of the accuracy or precision of rotation quantified by the offset of a flexure’s symmetry center, the flexibility quantified by strain energy and stress in elliptic flexure hinges, an FEM analysis is performed in terms of two non-dimensional parameters, which determines our geometry. Also we compared the accuracy of elliptic, parabolic, hyperbolic and circular flexure hinges by configuring the diagram including offset of flexure’s symmetry center vs. flexure's end deflection. The mentioned flexure hinge profiles are used in a sample mechanism and amplification factor, natural frequency and accuracy of results are compared invoking FEM results. In order to verify the obtained results, the sample
mechanism with corner-filletted flexure hinge with the fillet radius of 0.1 mm is manufactured and the FEM results is verified with measured displacement of this sample frame.

2. BASIC ASSUMPTIONS

The geometry equations of different types of flexure hinge considering Figure 1 and Figure 2 are:

**Figure 1: Parameters defining a symmetric conic-section flexure hinge**

<table>
<thead>
<tr>
<th>Equation Type</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right circular flexure hinge</td>
<td>( t(x) = t + 2[2r - \sqrt{x(2r - x)}] ) , ( r = c ), ( l = 2c )</td>
</tr>
<tr>
<td>Parabolic Flexure Hinge</td>
<td>( t(x) = t + 2\left( -\frac{x^2}{t} \right) )</td>
</tr>
<tr>
<td>Hyperbolic Flexure Hinge</td>
<td>( t(x) = \sqrt{x^2 + 4c(t + t\left( -\frac{x^2}{t} \right))} )</td>
</tr>
<tr>
<td>Elliptical Flexure Hinge</td>
<td>( t(x) = t + 2\left( 1 - \frac{1 - \left( -\frac{x^2}{t} \right)^2}{1} \right) )</td>
</tr>
</tbody>
</table>

**Figure 2. Cross-sectional profile of a symmetric corner-filletted flexure hinge**

Corner-Filleted Flexure Hinge:

\[
t(x) = \begin{cases} 
  t + 2\left[ r - \sqrt{x(2r - x)} \right], & x \in [0, r] \\
  t, & x \in [r, l - r] \\
  t + 2\left[ r - \sqrt{(l - x)(2r - (l - x))} \right], & x \in [l - r, r] 
\end{cases}
\]

Using these equations allows us to have a parametric model in FEM software, which provides us rapid change of flexure hinge types and geometric parameters. The geometric parameters are mentioned based on two non-dimensional parameters \( \beta \) and \( \gamma \), that are defined as:

\[
\beta = \frac{t}{2c}, \\
\gamma = \frac{t}{l}
\]

3. ACCURACY, FLEXIBILITY AND STRESS STUDY

a. Accuracy

The relative rotation of two mechanical members that are connected by a conventional rotation joint is produced along an axis that passes through the geometric center of the joint, which is fixed, provided one member is also fixed. In the case of a symmetric flexure hinge, the center of rotation (the geometric symmetry center of the flexure) is no longer fixed since the flexure is subjected to deformation.

In order to study the accuracy of flexure hinge, an FEM model is created in which the material used is steel \((E=200E9, \nu = 0.3)\). Figure 3) The displacement of the flexure hinge rotation center (point 2 in Figure 1) produced by input displacement of 0.1 mm applied in right edge, indicates the accuracy of flexure hinge. Whatever this displacement was smaller the flexure hinge operates closer to a real hinge. An elliptic flexure hinge with varying \( t/L \) and \( t/2c \) parameters \((L: constant)\) is studied according to Figure 4.

**Figure 3: Parametric design of an elliptic flexure hinge**
Different types of flexure with the same $t/L$ and $t/2c$ are analyzed in FEM software. The results are shown in Figure 5 and Figure 6.

From Figure 4, it can be seen that the accuracy of an elliptic flexure hinge decreases by increasing $t/L$ and $t/2c$.

Figure 5 and 6 show that hyperbolic flexure hinge is the most accurate profile and corner filleted flexure is the least accurate.

b. Flexibility

By calculating strain energy in FEM analysis, the flexibility of flexure hinge can be studied. The more flexible flexure hinge has the lower level of strain energy.

Figure 7 shows that bigger energy is consumed for deflection of flexure hinge with increasing in $t/L$ and decreasing in $t/2c$. It means that the flexure hinge with smaller $t/L$ and larger $t/2c$ has more flexibility.

c. Stress Study

The maximum von Mises stress, under specific constraint, which is happened in an elliptical flexure hinge, is showed in Figure 8.

Figure 8 shows that the maximum von Mises stress produced in an elliptical flexure hinge decreases using a flexure with smaller $t/L$ and larger $t/2c$.

It means that the flexure hinge with smaller $t/L$ and larger $t/2c$ has longer fatigue life.
Figure 8: The maximum von Mises stress for varying geometric parameters $t/L$ and $t/2c$.

4. MICRACTUATOR

A sample mechanism for microactuator is modeled in FEM software according to Figure 9 with the flexure hinge geometric specifications of $t = 0.3$, $c = 0.1$, $l = 1.5$ mm.

Figure 9: Parametric design of a flexure based microactuator

The amplification ratio and natural frequency of microactuator using different types of flexure hinge are in Table 1.

<table>
<thead>
<tr>
<th>Flexure Type</th>
<th>Amplification Ratio</th>
<th>Natural Frequency(Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corner Filleted</td>
<td>4.861448</td>
<td>218.79</td>
</tr>
<tr>
<td>elliptic</td>
<td>4.854049</td>
<td>237.27</td>
</tr>
<tr>
<td>parabolic</td>
<td>4.850139</td>
<td>244.75</td>
</tr>
<tr>
<td>hyperbolic</td>
<td>4.848669</td>
<td>247.20</td>
</tr>
</tbody>
</table>

The results in Table 1 indicate that the microactuator that uses corner filleted flexure hinge has the largest amplification ratio and lowest natural frequency. This means that such an actuator is suitable for displacement oriented applications that work in low frequencies.

In addition, it is apparent that the microactuator that uses hyperbolic flexure hinge is the stiffest structure.

5. EXPERIMENTAL VERIFICATION

The sample mechanism with corner-filleted flexure hinge with the fillet radius of 0.1 mm is manufactured according to Figure 10. The vertical displacement of microactuator end-effector is measured by a comparator. The experimental results are shown in Table 2.

Figure 10: A microactuator frame test under comparator

<table>
<thead>
<tr>
<th>Input</th>
<th>ANSYS Output</th>
<th>Experimental Output</th>
<th>Amplification Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.8614E-6</td>
<td>4.2E-06</td>
<td>4.86144</td>
</tr>
<tr>
<td>2</td>
<td>9.7292E-6</td>
<td>9.1E-06</td>
<td>4.86144</td>
</tr>
<tr>
<td>3</td>
<td>1.4584E-5</td>
<td>1.4E-05</td>
<td>4.86144</td>
</tr>
<tr>
<td>4</td>
<td>1.9445E-5</td>
<td>1.87E-05</td>
<td>4.86144</td>
</tr>
<tr>
<td>5</td>
<td>2.4307E-5</td>
<td>2.38E-05</td>
<td>4.86144</td>
</tr>
</tbody>
</table>

As it’s shown in Table 2 the experimental results are close to the FEM results. The differences between these results can be interpreted by manufacturing error and presence of an external force in experimental test.

6. CONCLUSION

Hyperbolic flexure hinge is the most accurate profile and corner filleted flexure is the least accurate.

A flexure hinge with smaller $t/L$ and larger $t/2c$ has more flexibility.

A flexure hinge with smaller $t/L$ and larger $t/2c$ has longer fatigue life.

A microactuator that uses corner filleted flexure hinge has the largest amplification ratio and lowest natural frequency.

A microactuator that uses hyperbolic flexure hinge is the stiffest structure.
REFERENCES